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Fixed point results for two pairs of non-self hybrid mappings in metric spaces of hyperbolic type

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ABSTRACT

This research paper proves some interesting results on common fixed point for two pairs of non-self hybrid (single valued and multivalued) contractive mappings in metric spaces of hyperbolic type. The results are established without employing the weakly commutativity and continuity assumptions. We adopted an existing method of proof to obtain our results. The results generalize and improve some results proved in related works in literature. An example is given to validate our claim.

1. Introduction

Fixed point theorem for single valued self-mappings in metric space was first proved by Banach (1992). Later Nadler (1969) introduced fixed point results for multivalued mappings in metric spaces. Takahashi (1970) introduced the property of convexity in metric spaces and established some fixed point theorems that generalized some results in Banach spaces. Assad and Kirk (1972) discovered that in convex metric spaces some maps are not self-mapping and proved the existence and uniqueness of the fixed point for non-self multivalued mapping in metric spaces. Kirk (1982) further introduced the concepts of metric spaces of hyperbolic type by placing Krasnoselskii's result (for $f_\lambda = (1-\lambda)I + \lambda I$ for some $\lambda \epsilon (0,1)$) in the setting of convex metric spaces.

Definition 1.1: Let (X,d) be a metric space where X is a non-empty set and d is a mapping $d: X \times X \to R$ such that for every $x, y, z \epsilon X$ (Frechet, 1906)

$d_1\ d(x,y) \geq 0,$
$d_2\ d(x,y) = 0$ if and only if $x = y,$
$d_3\ d(x,y) = d(y,x),$
$d_4\ d(x,z) \leq d(x,y) + d(y,z).$

Definition 1.2: Suppose X is a metric space and $R = [0,1]$ the closed unit interval. The convex structure on X is an operator $W: X \times X \times R \to X$ which satisfies the following axioms (Takahashi, 1970),

$$d(z, W(x,y,\beta)) \leq \beta\, d(z,x) + (1-\beta) d(z,y), \qquad (1.1)$$

for every $z \epsilon X$ and $\beta \epsilon R. If$ (X,d) is equipped with a convex structure, then X is known as convex metric space.

Definition 1.3: Let (X,d) be a metric space and L a family of metric segment. X is called a metric space of hyperbolic type if the following axioms are satisfied (Kirk, 1982);

(a) each two points $x, y \epsilon X$ are endpoints of exactly one number seg $[x,y]$ of L and,
(b)
$if\ u,x,y \epsilon X\ and\ z \epsilon\ seg\ [x,y]\ satisfies\ d(x,z) = \lambda\, d(x,y)\ for\ \lambda \epsilon [0,1]\ then$

$$d(u,z) \leq (1-\lambda) d(u,x) + \lambda\, d(u,y). \qquad (1.2)$$

Some authors worked on the convergence theorems of contractive maps in metric spaces and its generalizations with applications (Okeke and Abbas, 2015; Okeke and Kim, 2015; Bishop et al., 2017). Huang et al. (2014) established a common fixed point theorem for two pairs of non-self mappings satisfying certain generalized contractive conditions of Ciric type in cone metric spaces. Ahmed and Khan (1997) established the existence and uniqueness of some common fixed point of a pair of hybrid non-self mapping in metrically convex metric spaces. The authors in Ahmed and Khan (1997) gave the following definition.

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Definition 1.4: Let J be a non-empty closed subset of a metric space (X, d). Let $F: J \rightarrow CB(X)$ and $T: J \rightarrow X$. Then F is known as generalized T- contraction of J into $CB(X)$ if there exist non-negative real α, δ, γ with $\alpha + 2\delta + 2\gamma < 1$ such that for all $x, y \in J$

$$H(Fx, Fy) \leq \alpha\, d(Tx, Ty) + \delta\{d(Tx, Fx) + d(Ty, Fy)\} + \gamma\{d(Tx, Fy) + d(Ty, Fx)\}.$$

Ahmed and Imdad (1998) further generalized the result of Ahmed and Khan (1997) to two pairs of hybrid non-self-mappings in the same setting. Ciric and Cakić (2009) introduced new non-self contractive mappings and proved the coincidence and common fixed point for the two pairs of hybrid mappings in complete convex metric spaces. Ciric et al. (2007) established common fixed point theorems for two pairs of non-self hybrid operators fulfilling certain generalized contraction conditions without employing the compatibility and continuity of the mappings in metrically convex metric spaces. Eke (2016) proved the existence and uniqueness of common fixed point for a pair of weakly compatible non-self operators fulfilling more general contractive conditions in metric spaces of hyperbolic type. Eke et al. (2018) introduced a new class of nonlinear contraction operators in metric spaces and proved common fixed point theorem for a pair of non-self mappings fulfilling the new contraction conditions in metric spaces of hyperbolic type.

The purpose of this research is to prove the coincidence and common fixed point theorems for two pairs of non-self hybrid mappings fulfilling certain generalized contraction conditions in metric space of hyperbolic type.

2. Main results

Theorem 2.1: Suppose (X, d) is a metric space of hyperbolic type and K a nonempty closed subset of X. If δK is a nonempty boundary of $K, E, F: K \rightarrow CB(X)$ and $M, N: K \rightarrow X$ such that

$$H(Ea, Fb) \leq \alpha\, d(Na, Mb) + \beta\{d(Na, Ea) + Mb, Fb)\} + \gamma\{d(Na, Fb) + d(Mb, Ea)\}, \quad (2.1)$$

$for\ all\ a, b \in K\ where\ \alpha + \beta + \gamma < 1\ and\ \frac{\alpha+\beta+\gamma}{1-\beta-\gamma} < \frac{1}{2}$. If

(i) $\delta K \subseteq MK \cap NK, EK \cap K \subseteq NK, FK \cap K \subseteq MK$,

(ii) $Ma \in \delta K \Rightarrow Fa \subset K, Na \in \delta K \Rightarrow Ea \subseteq K$,

and $M(K)$ and $N(K)$ are complete then E and N have a coincidence, and F and M have a coincidence in K. Moreover if there exist u and w such that $Mu = Nw \in Ew = Fu$, then E, F, M and N have a common fixed point.

Proof: For an arbitrary $a \in \delta K$, we can develop three sequences $\{a_n\}$ and $\{c_n\}$ in K and $\{b_n\}$ in X. Assume $c_0 = a$. Since $c_0 \in \delta K$, there exists point $a_0 \in K$ such that $Ma_0 = Na_0 = c_0$. Now choose $c_0 = Ma_0$. We have $Ma_0 \in \delta K$ which implies that $Ea_0 \subseteq K$. Hence we conclude that $Ea_0 \subseteq K \cap EK$. From (i), $Ea_0 \subseteq NK$. Therefore there exists an $a_1 \in K$ such that $Na_1 \in Ea_0 \subset K$. Set $c_1 = b_1 = Na$. Since $b_1 \in Ea_0 \subset K$ and according to Nadler (Ahmed and Khan, 1997) there exists a point $b_2 \in Fa_1$ such that

$$d(b_1, b_2) \leq H(Ea_0, Fa_1) + \frac{(1-\beta-\gamma)\theta}{(1+\beta+\gamma)}.$$

Since $b_2 \in FK \cap K$, it follows that $b_2 \in MK$ by (i). Let $a_2 \in K$ such that $Ma_2 = b_2 = c_2 \in Fa_1$. If $b_2 \notin K$, then there exists $c_2 \in \delta K$ $(c_2 \notin b_2)$ such that $c_2 \in seg\,[b_1, b_2]$. Since $b_2 \in K$, then by (i) we have $Na_2 = c_2$.

This choice is possible because $c_2 \in \delta K \subseteq MK \cap NK$. Hence $c_2 \in \delta K \cap seg\,[b_1, b_2]$. We can choose $b_3 \in Fa_2 \subseteq K$ such that

$$d(b_2, b_3) \leq H(Ea_1, Fa_2) + \frac{(1-\beta-\gamma)\theta^2}{(1+\beta+\gamma)}.$$

Since $b_3 \in FK \cap K \subseteq MK$, there is a point $a_3 \in K$ such that $Ma_3 = b_3$.

Continuing in the process, we develop sequence $\{a_n\} \subseteq K$, $\{c_n\} \subseteq K$ and $\{b_n\} \subset MK \cup FK$ such that:

(a) $b_n \in Ea_{n-1}$ or $b_n \in Fa_{n-1}$;
(b) $c_n = Ea_n$ or $c_n = Na_n$;
(c) $b_n = c_n$ if and only if $b_n \in K$

and in this case; if $b_n \in Ea_{\{n-1\}}$ then $c_n = Fa_n$ and $b_{\{n+1\}} \in Fa_n$ is such that

$$d(b_n, b_{n+1}) \leq H(Ea_{n-1}, Fa_n) + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}.$$

or if $b_n \in Fa_{n-1}$ then $c_n = Na_n$ and $b_{n+1} \in Ea_n$ is such that

$$d(b_n, b_{n+1}) \leq H(Fa_{n-1}, Ea_n) + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}.$$

(d) $b_n \notin c_n$ whenever $c_n \in \delta K \cap seg[Ea_{n-2}, Ea_{n-1}]$. This proves that E, F, M and N are non-self-mappings.

Remarks 1: If $b_n \notin c_n$, then $c_n \in \delta K$. This implies that $c_{n+1} = b_{n+1} \in K$. Likewise, $c_{n-1} = b_{n-1} \in K$. If $c_{n-1} \in \delta K$ then it implies $c_n = b_n \in K$.

We can show that $\{c_n\}$ is a Cauchy sequence in K if $d(c_n, c_{n+1}) = 0$. The proof is trivial. Suppose $d(c_n, c_{n+1}) > 0$ for all n. We can consider three cases from (a), (b), (c) and (d) as follows;

(1) $c_n = b_n \in K$ and $c_{n+1} = b_{n+1}$
(2) $c_n = b_n \in K$ but $c_{n+1} \neq b_{n+1}$
(3) $c_n \neq b_n \in K$ implies

$c_n \in \delta K \cap seg[Ea_{n+2}, Ea_{n+1}]$.

Case 1: Let $c_n = b_n \in K$ and $c_{n+1} = b_{n+1}$. If $b_n \in Ea_{n-1}$, then $c_n = Na_n$, $Ma_{n-1} = c_{n-1}$, $b_{n+1} \in Fa_n$ and b_n and b_{n+1} are such that

$$d(b_n, b_{n+1}) \leq H(Ea_{n-1}, Fa_n) + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}.$$

Using equation (2.1) we obtain

$$d(b_n, b_{n+1}) \leq H(Ea_{n-1}, Fa_n) + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$$

$\leq \alpha\, d(Na_{n-1}, Ma_n) + \beta\{d(Na_{n-1}, Ea_{n-1}) + d(Ma_n, Fa_n)\} + \gamma\{d(Na_{n-1}, Fa_n) + d(Ma_n, Ea_n)\} + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$

$\leq \alpha\, d(c_{n-1}, c_n) + \beta\{d(c_{n-1}, b_n) + d(c_n, b_{n+1}\} + \gamma\{d(c_{n-1}, b_{n+1}) + d(c_n, b_n)\} + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$

Since $c_n = b_n \in K$ and $c_{\{n+1}= b_{n+1}$ for all $n \in N$, we get

$d(b_n, b_{n+1}) = d(c_n, c_{n+1}) \leq \alpha\, d(c_{n-1}, c_n) + \beta\{d(c_{n-1}, c_n) + d(b_n, b_{n+1})\} + \gamma\{d(c_{n-1}, c_n) + d(b_n, b_{n+1}\} + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$

$\leq (\alpha+\beta+\gamma)\, d(c_{n-1}, c_n) + (\beta+\gamma)\, d(b_n, b_{n+1}) + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$

$\leq \frac{(\alpha+\beta+\gamma)}{(1-\beta-\gamma)} d(c_{n-1}, c_n) + \frac{\theta^n}{(1+\beta+\gamma)}$ (2.2)

Case 2: Let $c_n = b_n \in K$ but $c_{n+1} \neq b_{n+1}$. This implies that $c_{n+1} \in \delta K \cap seg[b_n, b_{n+1}]$. From equation (1.2) with u = b, we get $d(b, c) \leq (1-\lambda)\, d(a, b)$.

Therefore, we obtain

$$d(a,b) \leq d(a,c) + d(c,b) \leq \lambda\, d(a,b) + (1-\lambda)\, d(a,b) = d(a,b).$$

Hence

$$c \in seg[a,b] \text{ implies } d(a,c) + d(c,b) = d(a,b)$$

because

$c_{n+1} \in seg[b_n, b_{n+1}] = seg[c_n, b_{n+1}]$. So

$d(c_n, b_{n+1}) = d(b_n, c_{n+1})$
$= d(b_n, b_{n+1}) - d(c_{n+1}, b_{n+1})$
$< d(b_n, b_{n+1})$

In view of (2.1) we obtain

$d(c_n, c_{n+1}) \leq \frac{(\alpha+\beta+\gamma)}{(1-\beta-\gamma)} d(c_{n-1}, c_n) + \frac{\theta^n}{(1+\beta+\gamma)}$

Case 3: *Let* $c_n \neq b_n$ *then* $c_n \in \delta K \cap seg[Ea_{n+1}, Ea_{n+1}]$ i.e. $c_n \in \delta K \cap seg[b_{n-1}, b_n]$.

By Remark 1 we get, $C_{n+1} = b_{n+1}$ *and* $c_{n-1} = b_{n-1}$. This implies that

$d(c_n, c_{n+1}) = d(c_n, b_{n+1}) \leq d(c_n, b_n) + d(b_n, b_{n+1}) = d(c_{n-1}, b_n) - d(c_{n-1}, c_n) + d(b_n, b_{n+1})$

$\leq d(b_{n-1}, b_n) + d(b_n, b_{n+1}).$ (2.3)

We shall find $d(b_{n-1}, b_n)$ and $d(b_n, b_{n+1})$. Since $c_{n-1} = b_{n-1}$, then we can conclude that

$d(b_{n-1}, b_n) \leq \frac{(\alpha+\beta+\gamma)}{(1-\beta-\gamma)} d(c_{n-2}, c_{n-1}) + \frac{\theta^{n-1}}{(1+\beta+\gamma)}$ (2.4)

with respect to case 2.

$d(b_n, b_{n+1}) \leq H(Ea_{n-1}, Fa_n) + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$

$\leq \alpha\, d(Na_{n-1}, Ma_n) + \beta\{d(Na_{n-1}, Ea_{n-1}) + d(Ma_n, Fa_n)\} + \gamma\{d(Na_{n-1}, Fa_n) + d(Ma_n, Ea_n)\} + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$

$\leq \alpha\, d(c_{n-1}, c_n) + \beta\{d(c_{n-1}, b_n) + d(c_n, b_{n+1}\} + \gamma\{d(c_{n-1}, b_{n+1}) + d(c_n, b_n)\} + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$

$\leq \alpha\, d(c_{n-1}, c_n) + \beta\{d(c_{n-1}, c_n) - d(c_n, b_n) + d(b_n, b_{n+1}) - d(c_n, b_n)\} + \gamma\{d(c_{n-1}, c_n) - d(c_n, b_{n+1}) + d(b_n, b_{n+1}) - d(c_n, b_{n+1})\} + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$

$\leq \alpha\, d(c_{n-1}, c_n) + \beta\{d(c_{n-1}, c_n) + d(b_n, b_{n+1})\} + \gamma\{d(c_{n-1}, c_n) + d(b_n, b_{n+1}) - d(c_n, b_{n+1})\} + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$

$\leq (\alpha+\beta+\gamma)\, d(c_{n-1}, c_n) + (\beta+\gamma)\, d(b_n, b_{n+1}) + \frac{(1-\beta-\gamma)\theta^n}{(1+\beta+\gamma)}$

$\leq \frac{(\alpha+\beta+\gamma)}{(1-\beta-\gamma)} d(c_{n-1}, c_n) + \frac{\theta^n}{(1+\beta+\gamma)}.$ (2.5)

Thus, in view of (2.4) and (2.5), we obtain

$d(c_n, c_{n+1}) \leq \frac{(\alpha+\beta+\gamma)}{(1-\beta-\gamma)} d(c_{n-1}, c_n) + \frac{(\alpha+\beta+\gamma)}{(1-\beta-\gamma)} d(c_{n-1}, c_{n-2}) + \frac{\theta^n}{(1+\beta+\gamma)} + \frac{\theta^{n-1}}{(1+\beta+\gamma)}$

$\leq 2\frac{(\alpha+\beta+\gamma)}{(1-\beta-\gamma)} \max\{d(c_n, c_{n-1}), d(c_{n-1}, c_{n-2})\} + \frac{\theta^n}{(1+\beta+\gamma)} + \frac{\theta^{n-1}}{(1+\beta+\gamma)}$

$\leq h \max\{d(c_n, c_{n-1}), d(c_{n-1}, c_{n-2})\} + \frac{\theta^n}{(1+\beta+\gamma)} + \frac{\theta^{n-1}}{(1+\beta+\gamma)}$ (2.6)

where

$h = \frac{(\alpha+\beta+\gamma)}{(1-\beta-\gamma)} < \frac{1}{2}.$

In view of equation (2.2) and (2.6) we get

$$d(c_n, c_{n+1}) = \begin{cases} \frac{\alpha+\beta+\gamma}{1-\beta-\gamma} d(c_n, c_{n-1}) + \frac{\theta^n}{1+\beta+\gamma} \\ h \max\{d(c_{n-2}, c_{n-1}), d(c_{n-1}, c_n)\} + \frac{\theta^n}{1+\beta+\gamma} + \frac{\theta^{n-1}}{1+\beta+\gamma} \end{cases}$$

According to Itoh (1977) can be shown that the sequence $\{c_n\}$ is Cauchy. Since K is closed, then it has a limit point say c ϵ K such that $\lim_{n\to\infty} c_n = c$.

Next we show that $c \in M(K) \cap N(K)$. First if two subsequences $\{c_{nj}\}$ and $\{c_{nk}\}$ are defined by $\{c_{nj}\} = Ma_{nj} \in Fa_{nk}$ and *by* $c_{nk} = Na_{nk} \subseteq Ea_{nk-1}$, respectively are infinite. Then

$\lim_{j\to\infty} Ma_{nj} = c$ and $\lim_{k\to\infty} Na_{nk} = c$.

Since $M(K)$ and $N(K)$ are complete then we have $c \in M(K) \cap N(K)$.

If one of the subsequences $\{c_{nj}\}$ or $\{c_{nk}\}$ is finite. Then there is an infinite subsequence $\{c_{nm}\}$ of $\{c_n\}$ such that $\{c_{nm}\} \in \delta K$. Since $\{c_{nm}\} \in \delta K$ and $\{c_{nm}\} \to c$ as $n \to \infty$ then it implies that $c \in \delta K$. Hence by (i) of Theorem 2.1, $c \in M(K) \cap N(K)$. Thus we have shown that $c \in M(K) \cap N(K)$.

It follows that there are some $u, w \in K$ such that $Mu = c = Nw$.

Now, we show that w is the coincidence point of E and N and that u is the coincidence point of F and M. Since $\{c_n\} = \{c_{nj}\} \cup \{c_{nk}\}$, where the subsequence $\{c_{nj}\}$ and $\{c_{nk}\}$ are defined as above; if one of them is infinite and without lost of generality, let $\{c_{nj}\}$ be infinite, where $\{c_{nj}\} = Ma_{mj} = b_{mj} \in Fa_{nj-1}$ and, using (2.1) we have

$d(Ew, c_{nj}) \leq H(Ew, Fa_{nj-1}) + \frac{\theta^{nj}}{(1+\beta+\gamma)}$

$\leq \alpha\, d(Nw, Ma_{nj-1}) + \beta\{d(Nw, Ew) + d(Ma_{nj-1}, Fa_{nj-1})\} + \gamma\{d(Nw, Fa_{nj-1}) + d(Ma_{nj-1}, Ew)\}$
$+ \frac{\theta^{nj}}{(1+\beta+\gamma)}$
$= \alpha d(c, c_{nj}) + \beta\{d(c, Ew) + d(c_{nj-1}, c_{nj}\} + \gamma\{d(c, c_{nj}) + d(c_{nj}, Ew)\} + \frac{\theta^{nj}}{(1+\beta+\gamma)}$

$As j \to \infty$ we obtain,

$d(Ew, c) \leq (\beta + \gamma) d(c, Ew) = (\beta + \gamma) d(Ew, c).$
$(\beta - \gamma)\, d(c, Ew) \leq 0.$

But $(1 - \beta - \gamma) > 0$. Hence $d(c, Ew) \leq 0$. Since metric is nonnegative we have $d(Ew, c) = 0$. Thus $c \epsilon Ew$ as E is closed. Thus $Nw \epsilon Ew$. Similarly, from (2.1) we have,

$$d(c, Fu) \leq H(Ew, Fu)$$
$$\leq \alpha d(Nw, Mu) + \beta\{d(Nw, Ew) + d(Mu, Fu)\} + \gamma\{d(Nw, Fu) + d(Mu, Ew)\}$$
$$\leq \beta d(c, Fu) + \gamma\, d(c, Fu)$$
$$\leq (\beta + \gamma) d(c, Fu),$$

a contradiction, hence $d(c, Fu) = 0$. Therefore $c = Fu$. Thus we have $Mu \epsilon Fu$.

Thus c is the point of coincidence for E, N and also for F, M.

Similarly, we have $d(Ew, Fu) = 0$. Hence $Ew = Fu$. Therefore we have proved that $Mu = Nw \epsilon Ew = Fu$. Thus, E, F, M and N have a common fixed point.

If $E = F$ and M =N in Theorem 2.1 then we obtain the following Corollary and the proof follows as well.

Corollary 2.2: Let (X, d) be a metric space of hyperbolic type and K a nonempty closed subset of X. If δK is nonempty and δK be the boundary of K and $F: K \rightarrow CB(X)$ and $N: K \rightarrow X$ such that

$$H(Fa, Fb) \leq \alpha\, d(Na, Nb) + \beta\{d(Na, Fa) + d(Nb, Fb)\} + \gamma\{d(Na, Fb) + d(Nb, Fa)\}$$

for all a, b ϵ K where α, β, γ are non -negative real numbers such that $\frac{(\alpha + \beta + \gamma)}{(1 - \beta - \gamma)} < 1$. If

(i) $\delta K \subseteq NK, FK \subseteq NK,$
(ii) $Na \epsilon \delta K \Longrightarrow Fa \subset K,$

and N(K) is complete then F and N have a coincidence in K.

Remark 2.3: Theorem 2.1 is proved in the setting of metric spaces of hyperbolic type without compatibility and continuity of the functions. Thus, Theorem 2.1 generalized Theorem 3.1 of Ahmed and Khan (1997). Theorem 2.1 is independent of Theorem 2.1 of Ciric et al. (2007) in the setting of metric spaces of hyperbolic type.

Example 2.4: Let $X = [0, +\infty)$ be defined with the usual metric and $K = [0, 2]$. Let $E, F: K \rightarrow X$ and $M, N: K \rightarrow CB(X)$ be defined by $Ea = \frac{a^2}{2}$, $Fa = a$, $Ma = 2a$ and $Na = a^2$, $\delta K = \{0, 2\}$.
begin{eqnarray*}
$d(Ea, Fb) = |\frac{a^2}{2} - b| = |\frac{a^2-2b}{2}| = \frac{1}{2}|a^2 - 2b| \leq \alpha\, d(Na, Mb).$

This satisfies (ii) of Theorem 2.1 and (2.1) with $\alpha = \frac{1}{2}$ and $\beta = \gamma = 0$. Let $a \epsilon K$ and $b \notin K$ then there exist $c = 2 \epsilon \delta K$ and $2 \epsilon Seg[2, 3]$ such that $2 \epsilon \delta K \cap Seg[2, 3]$.

(i) $\{0, 2\} \subseteq [0, 4] \cap [0, 4], [0, 2] \cap [0,2] \subseteq [0, 4], [0, 2] \cap [0,2] \subseteq [0, 4].$
(ii) $M(0) \epsilon \delta K \Longrightarrow F(0) \subseteq K, N(0) \epsilon \delta K \subseteq E(0) \subseteq K.$

Thus all the conditions of Theorem 2.1 is satisfied and E and N have a coincidence, F and M have a coincidence in K.

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